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Lemons and Money Markets*

Abstract. This paper identifies simple conditions for monotone comparative statics of a unique equilibrium in the Akerlof-Wilson model. Separate conditions apply to trade volume and price. Trade volume increases when supply becomes both stronger and *more* elastic. In contrast, price decreases when supply becomes both stronger and *less* elastic. An application to the interbank market suggests surprisingly specific measures to address elevated term rates and market breakdown.

Keywords. Adverse selection; uniqueness of equilibrium; monotone comparative statics; elasticity of supply; log-supermodularity; log-concavity; interbank markets

JEL-Codes. D82, G21, G01

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1 Introduction

Competitive markets with asymmetric information, as described first by Akerlof (1970), have become a cornerstone of the literature on adverse selection. A population of sellers, privately informed about the quality of their respective endowment, meets a population of uninformed, but otherwise interested buyers. A seller offers her endowment only if the market price exceeds the endowment's true value. The buyers, however, anticipate the self-interested behavior of sellers, and are willing to pay only for the average quality in the market. Through this restriction, the buyers' rational assessment of quality leads to a feedback loop that makes the behavior of such markets for "lemons" differ structurally from the Walrasian model with symmetric information. In particular, the market may break down completely. Moreover, as shown by C. Wilson (1979, 1980), in such markets the law of demand need not hold in general, giving rise to the possibility of multiple competitive equilibria, where an equilibrium with a lower price is Pareto dominated by an equilibrium with a higher price. But the equilibrium is known to be unique, and in fact constrained efficient, provided the distribution of quality on the supply side is not too convex (cf. Bigelow, 1990). Assuming uniqueness of the equilibrium, we study the consequences of changing the distribution of quality.

A number of papers has looked at comparative statics properties of markets for "lemons". Genesove (1993) shows that car sellers with a higher propensity to sell obtain a premium in the wholesale auction market. The underlying model is an extension of the Akerlof-Wilson framework. Sellers own several units, but can keep only a limited number on stock, which can then be sold at face value to informed consumers. Compared to the basic model, the extension imposes an additional constraint on the seller's problem. A higher propensity to sell is tantamount to a tighter capacity constraint in stock management, hence to a higher average quality of wholesale supply coming from that car seller. Thus, while similar at first sight, Genesove's insights cannot be used to obtain predictions about the comparative statics of the equilibrium in the basic model. In Gibbons and Katz

(1991), lower wages are predicted for laid-off workers than for workers unemployed as a result of a plant closing. Again, the exogenous change concerns the constraints on the problem of the supply side, not the underlying distribution of the quality parameter. Greenwald (1986) shows that an increase of random quits of high-quality workers, which add to the population of low-quality workers in the market, both increases second-hand wages and has a multiplier effect on job turnover. Moreover, a mean-preserving spread in the ability distribution lowers the second-hand wage. Comparative statics properties have been studied also with respect to the information structure. Kessler (2001) considers an extension with a fraction of uninformed sellers. It is shown that market performance is non-monotonic in the number of uninformed sellers. See also Levin (2001), who studies likewise the consequences of changing the information structure in the Akerlof-Wilson model. In sum, it appears to us that the intuitive properties of the Akerlof-Wilson model reported in this paper might have been overlooked so far, maybe due to the lack of a relevant application.

In this paper, the quality distribution of sellers changes into a new distribution which is stochastically dominated by the initial distribution. As a result of the logic of adverse selection, supply will then be stronger for any given market price. However, due to the endogenous formation of beliefs about quality in the market, the consequences on demand are in general ambiguous. As a consequence, both trade volume and price may either rise or fall in a market for "lemons". Our contribution is the identification of simple sufficient conditions for monotone comparative statics. We look first at volumes. It is shown that if the change in the quality distribution involves that supply becomes both stronger and everywhere *more elastic*, then trade volumes must increase. However, prices can move either way. Next, we look at the comparative statics of prices. Interestingly, the corresponding result for prices needs a different set of assumptions. Specifically, we show that if the change in the quality distribution involves that supply becomes both stronger and *less elastic*, then the market price must decrease, while no predictions can be made for the volume of trade. In addition to the comparative statics results for volume and price, we also offer a new proof of uniqueness of equilibrium, based on two alternative conditions on the primitives of the model that cover a large class of statistical distributions.

There are real markets where simple comparative statics properties do matter. Consider, for example, the interbank money market. Under normal conditions, an increase in the demand for liquidity might simply increase trading volume. But under adverse selection, a similar development might lead to a market breakdown, as has been the case in many currency areas during the 2007-2009 liquidity turmoil. The results of our theoretical analysis can be used to better understand the conditions under which a given market behaves one way or the other.

Our analysis will focus on the static model of adverse selection because uniqueness of the dynamic equilibrium is much harder to achieve. But we recognize that dynamic aspects can be quite important if additional assumptions on durability and contracting possibilities are satisfied.[1]

The remainder of this paper is organized as follows. In Section 2, we introduce the model, and discuss uniqueness of the competitive equilibrium. Sufficient conditions for monotone comparative statics of trade volume and price are derived in Section 3. The case of heterogeneous sellers is dealt with in Section 4. Section 5 contains an application to interbank markets. Section 6 concludes. The Appendix contains an intuitive auxiliary result.

2 "Lemons" markets with a unique equilibrium

We consider a competitive market for a commodity that varies in quality. Information about quality is asymmetrically distributed. Sellers know the quality of their respective endowment. Buyers, however, can only observe the price, not quality. Throughout the paper, we exclude the emergence of signaling conventions, i.e.,

[1] See, for instance, Kim (1985) and Hendel and Lizzeri (1999).

sellers cannot invest in observable characteristics which would distinguish their commodities from those of lower quality. This assumption is plausible, in particular, in our later example of distressed interbank markets (cf. Section 5).

On the supply side, there is a continuum of atomistic sellers. Each seller is endowed with a single unit of the commodity and maximizes the utility function

$$U = U(n, c \mid q) = nq + c, \tag{1}$$

where $n = 0, 1$ denotes a dummy variable representing consumption of the commodity, q represents quality, and c is consumption of other goods. For expositional reasons, we focus initially on the case where sellers have homogeneous preferences over quality. The analysis of the case of heterogeneous sellers is deferred to Section 4.

Quality is distributed on some interval $[q_0, q_1]$ with $q_0 < q_1$. The density function of quality, assumed to be continuous differentiable and strictly positive on $[q_0, q_1]$, is denoted by f. Let $F(p) = \int_{q_0}^{p} f(q)dq$ be the mass of sellers whose endowment has quality $q \leq p$. To be able to compare across markets, we allow for the possibility that $F(q_1)$ differs from unity. At market price p, a seller offers her endowment in the market if and only if $q \leq p$, with indifference when $q = p$. Thus, *supply* at price p is given by[2]

$$S(p) = \begin{cases} \int_{q_0}^{p} f(q)dq & \text{for} \quad p > q_0 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

We can determine the average quality offered in the market provided that supply does not vanish. Since the density f is strictly positive on $[q_0, q_1]$, this is the case

[2] In fact, our normalizations imply an identity of $F(p)$ and $S(p)$.

for prices $p > q_0$. Hence, average quality at price p reads

$$\bar{q}(p) = E[q \mid q \leq p] = \begin{cases} \frac{\int_{q_0}^{p} q f(q) dq}{S(p)} & \text{for } \ p > q_0 \\ q_0 & \text{for } \ p = q_0, \end{cases} \tag{3}$$

where the boundary case $p = q_0$ is the result of a continuous extension.

On the demand side, there is a continuum of atomistic buyers. Buyers know the distribution of quality F and correctly infer average quality $\bar{q}(p)$ from the observed market price p. Each buyer maximizes expected utility

$$U^e = U^e(n, c \mid t, p) = nt\bar{q}(p) + c, \tag{4}$$

where t measures the buyer's relative valuation of quality in terms of other consumption. The buyer's type t is drawn from some interval $[t_0, t_1]$, where $t_0 < t_1$. The density function of the relative valuation t is denoted by h. We assume that h is continuous, but not necessarily positive on $[t_0, t_1]$. Write $H(t) = \int_{t_0}^{t} h(\tau) d\tau$ for the mass of buyers with a type $\tau \leq t$, where again, we do not require that $H(t_1)$ be unity. A buyer of type t rationally purchases at price p provided that

$$t \geq \frac{p}{\bar{q}(p)}, \tag{5}$$

with indifference when (5) holds with equality. The term $p/\bar{q}(p)$ will be called the *price-quality ratio* in the sequel. It is well-defined provided $p \geq q_0$ and $\bar{q} \neq 0$. The function H contains information about the "willingness to buy" at a given price-quality ratio, and we will make use of this interpretation. Incorporating the buyers' anticipation of average quality offered in the market, *Walrasian demand* at price p is given by

$$D(p) = \begin{cases} \int_{p/\bar{q}(p)}^{t_1} h(t) dt & \text{for } \ p < t_1 \bar{q}(p) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Much of our subsequent analysis will be concerned with the question of how Walrasian demand changes in response to an exogenous change in the distribution of quality on the supply side.

To have a simple definition of the competitive equilibrium, we will consider only equilibria for which average quality is well-defined.[3] Consequently, we may define a *competitive equilibrium* as a pair (p^*, S^*) consisting of a price $p^* \geq q_0$ and a volume S^* such that $S(p^*) = S^* = D(p^*)$. Existence is straightforward. Indeed, at price $p = q_0$, supply is zero, while at price $p = \max\{t_1 q_1; q_0\}$, demand is nil. Therefore, existence of the competitive equilibrium follows from the continuity and non-negativity of supply and Walrasian demand.

Wilson (1979) constructed an example that illustrates the possibility of multiple equilibria in the above model. Specifically, multiplicity may, but need not, occur when the supply elasticity of average quality with respect to price exceeds one, i.e., when

$$\overline{\varepsilon}(p) = \frac{\partial \bar{q}(p)}{\partial p} \frac{p}{\bar{q}(p)} > 1 \tag{7}$$

for some price level p. In this case, an increase of the price by a percentage point makes average quality increase by more than one percentage point so that the price-quality ratio actually decreases. As a consequence of this more attractive situation for the buyers, Walrasian demand is actually increasing, so that multiple equilibria may arise. Wilson's example is reproduced on the left-hand panel of Figure 1.

However, as noted by Rose (1993), the condition for an increasing Walrasian demand and thus, potential multiplicity of equilibria, is unlikely to be satisfied when the quality distribution has a standard form. Further below we will actually strengthen Rose's numerical findings and verify also analytically that Walrasian demand is downward-sloping in the Akerlof-Wilson model for a large class of sta-

[3]This is actually without loss of generality. To understand why, note that a price strictly below q_0 could only belong to an equilibrium with rational expectations if $t_1 \leq 1$, which means there are no gains from trade. This case, however, would be without interest.

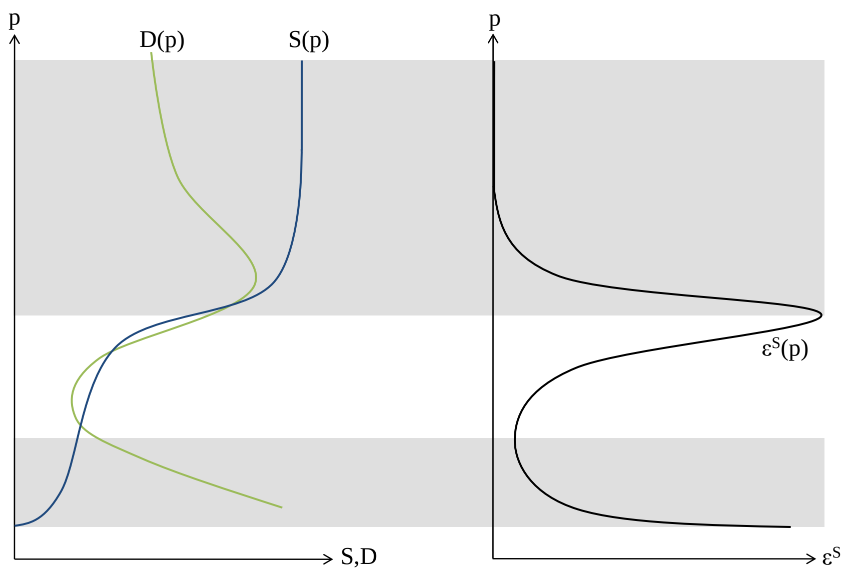

Figure 1: Supply, demand, and elasticity of supply in Wilson's (1979) example.

tistical distributions of quality. In fact, because the supply curve in (2) is strictly increasing in price over the support of the quality distribution, a weakly decreasing Walrasian demand is sufficient for uniqueness of the competitive equilibrium. Concerning the empirical perspective, Rose's results suggest that the restriction to markets with weakly decreasing demand is a relatively mild restriction.

A simple and intuitive sufficient condition for a weakly decreasing Walrasian demand and thus, for a unique equilibrium, can be formulated in terms of the *price elasticity of supply*, defined as

$$\varepsilon^S(p) = \frac{\partial S(p)}{\partial p} \frac{p}{S(p)} \tag{8}$$

for prices $p \geq q_0$. This price elasticity of supply should not be confused with the supply elasticity of average quality with respect to price (7), which plays in fact a related role in the Akerlof-Wilson model, as we have discussed above. The following condition is key to all our subsequent results concerning uniqueness and monotone comparative statics.

(NIES) $\partial \varepsilon^S(p)/\partial p \leq 0$ for all $p \in (q_0, q_1)$.

This condition requires that the price elasticity of supply is non-increasing in the market price. The result below follows essentially from Bigelow (1990).[4] We offer, however, a much crisper proof.

Theorem 1. Assume that condition (NIES) holds in a market with homogeneous sellers. Then $\partial D(p)/\partial p \leq 0$ for all $p \in (q_0, q_1)$. Moreover, the competitive equilibrium is unique.

Proof. We show first that demand is weakly downward-sloping on the support of the quality distribution. By equation (6), it suffices to show that $p/\bar{q}(p)$ weakly increases in p, i.e. that

$$\frac{\partial}{\partial p}\left[\frac{pF(p)}{\int_{q_0}^{p} qf(q)dq}\right] \geq 0 \tag{9}$$

for all $p \in (q_0, q_1)$. But inequality (9) is equivalent to

$$\frac{\partial}{\partial p}\left[\frac{\int_{q_0}^{p} qf(q) + F(q)dq}{\int_{q_0}^{p} qf(q)dq}\right] \geq 0, \tag{10}$$

which is nothing but a single crossing condition in monotone comparative statics under uncertainty (cf., e.g., Athey, 2002). To understand why, write

$$g(q, a) = qf(q) + aF(q), \tag{11}$$

[4] To see this, align the models by letting $a^s(0) = a^b(0) = 0$. Then, the right-hand side of equation 17.1 in Bigelow's paper equals $1/\theta$, yielding exactly the condition we impose in Theorem 1.

where $a = 0, 1$. Then inequality (10) says that

$$G(p, a) = \int_{q_0}^{p} g(q, a)dq \tag{12}$$

is log-supermodular in (p, a). But this property is preserved under integration, hence a sufficient condition for (10) is the log-supermodularity of g, i.e.,

$$\frac{\partial}{\partial p}\left[\frac{pf(p) + F(p)}{pf(p)}\right] \geq 0. \tag{13}$$

Rewriting (13) yields

$$\frac{\partial}{\partial p}\left[\frac{d \log F(p)}{d \log p}\right] \leq 0 \tag{14}$$

as a sufficient condition for Walrasian demand in the Akerlof-Wilson model to be weakly downward-sloping over the interval $[q_0, q_1]$. But this is just condition (NIES). To prove uniqueness of the competitive equilibrium, recall that $f(q) > 0$ for any $q \in [q_0, q_1]$. Consequently, supply $S(p)$ is strictly increasing on the interval $[q_0, q_1]$, while constant and strictly positive for $p > q_1$. By the first part of the theorem, Walrasian demand $D(p)$ is weakly declining on the interval $[q_0, q_1]$, and strictly declining or nil for $p > q_1$. Hence, excess demand at price p, i.e., the difference $D(p) - S(p)$, is strictly decreasing or negative for all $p \geq q_0$. It follows that there is at most one competitive equilibrium. This proves the theorem. $\square$

Intuitively, the crucial point in the buyers' decision is the price-quality ratio offered in the market. If the price elasticity of supply is *everywhere* monotonically non-increasing in price, then a one percent increase in price will change the average quality by no more than one percent. Hence, Walrasian demand is weakly declining under this condition.

Bigelow (1990) obtained a much more involved condition for uniqueness, which implies constrained ex-ante efficiency, and manages to interpret it as a restriction on the convexity of the quality distribution. More specifically, he pointed out that his condition is equivalent to the requirement that the Arrow-Pratt coefficient of absolute risk aversion of the "utility function" $\log F(q)$ is bounded by a certain ratio involving the quality parameter. Obviously, our condition of non-increasing price elasticity of supply is much simpler, which is striking. The reason for the complexity of Bigelow's condition is that it allows for a *benefit from owning the commodity even if quality is zero.* Assuming this benefit is zero for both seller and buyer, which is in fact standard in the Akerlof-Wilson model (cf., e.g., the utility specification in Wilson, 1980), we obtain our more transparent condition for uniqueness.

In Wilson's example, the price elasticity condition (NIES) and hence, Theorem 1, are not satisfied. As the right-hand panel of Figure 1 shows, the elasticity of supply is declining at low and high price levels (grey area), but increasing at intermediate price levels (white area). The strong increase of the elasticity of supply has the effect that a marginal increase in the price leads to a strong inflow of additional sellers offering relatively high quality. Thus, price is a very sensitive indicator for average quality at intermediate price levels, so that there is a domain where Walrasian demand actually increases with the price.[5]

The question arises how likely it is that the elasticity condition (NIES) will be satisfied. As mentioned before, Rose (1993) has discussed the question of uniqueness of equilibrium numerically, with the conclusion that a large number of standard statistical quality distributions imply a unique equilibrium. We can strengthen those numerical results with the following analytical proposition.

Theorem 2. Assume that either (i) the distribution function F is log-concave in $\log q$ or (ii) the density f is continuously differentiable and log-concave in $\log q$.[6] Then, $\partial\varepsilon^S(q)/\partial q \leq 0$ for all $q > q_0$.

Proof. We show first that (ii) implies (i). Assume that $f(q)$ is log-concave in

[5] As Figure 1 illustrates, Walrasian demand may even continue to increase when the elasticity of supply already declines. The reason for that delayed phase of Walrasian demand is the familiar feature of the Akerlof model that demand averages over all submarginal qualities.

[6] When f is smooth, then the latter condition reads $qf(q)f''(q)+f'(q)f(q)-qf'(q)^2 \leq 0$.

$\log q$. Write $\pi = \log q$. Then, $q = \exp(\pi)$, and

$$\log f(q) = \log f(\exp(\pi)) \tag{15}$$

is concave in π. Write

$$g(\pi) = f(\exp(\pi)) \exp(\pi). \tag{16}$$

From

$$\log g(\pi) = \log f(\exp(\pi)) + \pi \tag{17}$$

it follows that $\log g(\pi)$ is concave in π. Hence, $g(\pi)$ is log-concave in π. By Theorem 1 in Bagnoli and Bergström (2006), the indefinite integral $G(\pi) = F(\exp(\pi))$ is therefore also log-concave in π. Thus $F(q)$ is log-concave in $\log q$. To prove that condition (i) implies $\partial\varepsilon^S(q)/\partial q \leq 0$, note that

$$\frac{\partial\varepsilon^S(p)}{\partial q} = \frac{\partial}{\partial q}\left[\frac{\partial \log F(q)}{\partial \log q}\right] = \frac{1}{q}\left[\frac{\partial^2 \log F(q)}{\partial(\log q)^2}\right] \leq 0. \tag{18}$$

Hence, the assertion. □

From Theorem 2 it follows that numerous standard distributions feature a non-increasing elasticity of supply. Specific examples include the uniform distribution, the log-normal distribution, the half-normal distribution, the exponential distribution, the gamma distribution, the chi-squared distribution, the chi distribution, the F distribution, the Student's t, the Pareto distribution, the Maxwell distribution, and the Rayleigh distribution. All these quality distributions allow at most one equilibrium.[7]

3 Comparative statics: volumes and prices

Provided that condition (NIES) and, hence, Theorem 1 holds, the competitive

[7]In particular, with the help of our sufficient conditions, we can analytically confirm all the declining-demand results obtained numerically by Rose (1993).

equilibrium is unique, so that a comparative statics analysis is feasible without further preparation. The objective of this section will be to predict how trade volume and price of the unique competitive equilibrium will move in response to a change in the quality distribution, i.e., we will seek conditions that guarantee a higher or lower volume and a higher or lower price, respectively, in response to an exogenous change on the supply side. In fact, with a view on the application in Section 5, and also because it does not complicate things, we will also allow for a change in the distribution of buyers' preferences.

Consider two independent markets A and B, each with the characteristics described in the previous section. By expanding type intervals, if necessary, we may assume without loss of generality that the interval $[t_0, t_1]$ for preference parameters on the demand side is common to markets A and B. We will envisage throughout a scenario where the exogenous distribution of qualities in market B first-order stochastically dominates the corresponding distribution in market A, so that by the logic of adverse selection, supply will be stronger in market A than in market B, for any given price. I.e., we assume $F_A(p) \geq F_B(p)$ for all p. To avoid complications, we assume that support intervals for the quality parameter are identical and in the positive domain. I.e. the quality distributions in markets A and B have support $[q_0, q_1]$.

We first turn to the comparative statics of equilibrium volumes. It will be shown that a stronger and more elastic supply, combined with a stronger "willingness to buy" leads to a higher trade volume. It will also be shown that the elasticity condition is essential for this conclusion, and that prices may either rise or fall under these conditions.

Formally, the elasticity condition says that the elasticity of supply $\varepsilon_A^S(p) = \partial \log S_A(p)/\partial \log p$ in market A should be higher than or equal to the elasticity of supply $\varepsilon_B^S(p) = \partial \log S_B(p)/\partial \log p$ in market B for all $p \in (q_0, q_1)$.[8] To formalize a

[8]The two assumptions on the distribution of quality are not in conflict. See Example 2.

higher "willingness to buy" in market A, we assume that the mirror-image of the buyers' type distribution in market B stochastically dominates the corresponding distribution in market A, i.e., we assume

$$\int_t^{t_1} h_A(\tau)d\tau \geq \int_t^{t_1} h_B(\tau)d\tau \tag{19}$$

for all $t \leq t_1$. This latter requirement implies that for any given price-quality ratio, demand will be at least as large in market A as in market B.[9]

The result below gives sufficient conditions for equilibrium volumes to increase in a "lemons" market.

Theorem 3. In the model with homogeneous sellers, if

(i) Condition (NIES) holds in both markets,

(ii) $F_A(q) \geq F_B(q)$ for all q,

(iii) $\varepsilon_A^S(p) \geq \varepsilon_B^S(p)$ for all $p > q_0$, and

(iv) $\int_t^{t_1} h_A(\tau)d\tau \geq \int_t^{t_1} h_B(\tau)d\tau$ for all $t \leq t_1$,

then $S_A^* \geq S_B^*$, where S_i^* denotes the respective trade volume in the unique equilibrium in market $i = A, B$.

Proof. Let p_A^* and p_B^* be the respective prices in the unique competitive equilibria of markets A and B. To prove the theorem, we consider two cases. Assume first that $p_A^* \geq p_B^*$. Then, using the fact that F_A is monotonically increasing, as well as condition (ii) from the statement of the theorem,

$$S_A(p_A^*) = F_A(p_A^*) \geq F_A(p_B^*) \geq F_B(p_B^*) = S_B(p_B^*). \tag{20}$$

[9] Somewhat paradoxically, Walrasian demand may still be weaker in market A than in market B under this condition. Example 1 below, with the roles of markets A and B exchanged, captures this possibility.

Hence, $S_A^* \geq S_B^*$ in this case. Assume now that $p_A^* < p_B^*$. From condition (i) in the statement of the theorem, we find $D_A(p_A^*) \geq D_A(p_B^*)$. In analogy to the first case, it suffices to show that $D_A(p_B^*) \geq D_B(p_B^*)$, or equivalently, that

$$H_A(t_1) - H_A(\frac{p_B^*}{\bar{q}_A(p_B^*)}) \geq H_B(t_1) - H_B(\frac{p_B^*}{\bar{q}_B(p_B^*)}), \tag{21}$$

where $\bar{q}_A(p)$ and $\bar{q}_B(p)$ denote average qualities in markets A and B, respectively. Given condition (iv) in the statement of the theorem, it suffices to show that

$$\frac{p}{\bar{q}_A(p)} \leq \frac{p}{\bar{q}_B(p)} \tag{22}$$

for all $p > q_0$. By Lemma A.1 in the Appendix, the comparison of (22) is implied by $\varepsilon_A(p) \geq \varepsilon_B(p)$ for $p > q_0$. This proves the assertion. $\square$

Intuitively, when supply is everywhere more elastic in market A than in market B, then the price-quality ratio $p/\bar{q}_A$ in market A never exceeds the price-quality ratio $p/\bar{q}_B$ in market B. Therefore, demand in market A is everywhere stronger than in market B. Thus, ceteris paribus, equilibrium trade volume is increasing with a stronger and more elastic supply. A concomitant change to a higher "willingness to buy" does not impair this conclusion. We will now demonstrate the importance of the elasticity condition for obtaining the conclusion of Theorem 3.

Example 1. Consider the cumulative distribution function $F(q) = q^\varepsilon$ on $[0, 1]$, where $\varepsilon > 0$ is the constant elasticity of supply. Average quality at price $p \geq 0$ reads $\bar{q}(p) = p\varepsilon/(1 + \varepsilon)$. Walrasian demand is perfectly inelastic in this market, with

$$D(p) = \int_{(1+\varepsilon)/\varepsilon}^{t_1} h(t)dt. \tag{23}$$

Equating supply and demand, one obtains the equilibrium price

$$p^* = \left(\int_{(1+\varepsilon)/\varepsilon}^{t_1} h(t)dt \right)^{\frac{1}{\varepsilon}}. \tag{24}$$

Thus, trade volume is increasing in ϵ. The comparative statics of this example is illustrated in Figure 2 for parameter values $\varepsilon_A = 0.8$ and $\varepsilon_B = 1.5$, where the "willingness to buy" is distributed identically across markets and uniformly on $[t_0, t_1] = [1.8, 2.8]$. One can easily see that $S_A^* < S_B^*$.

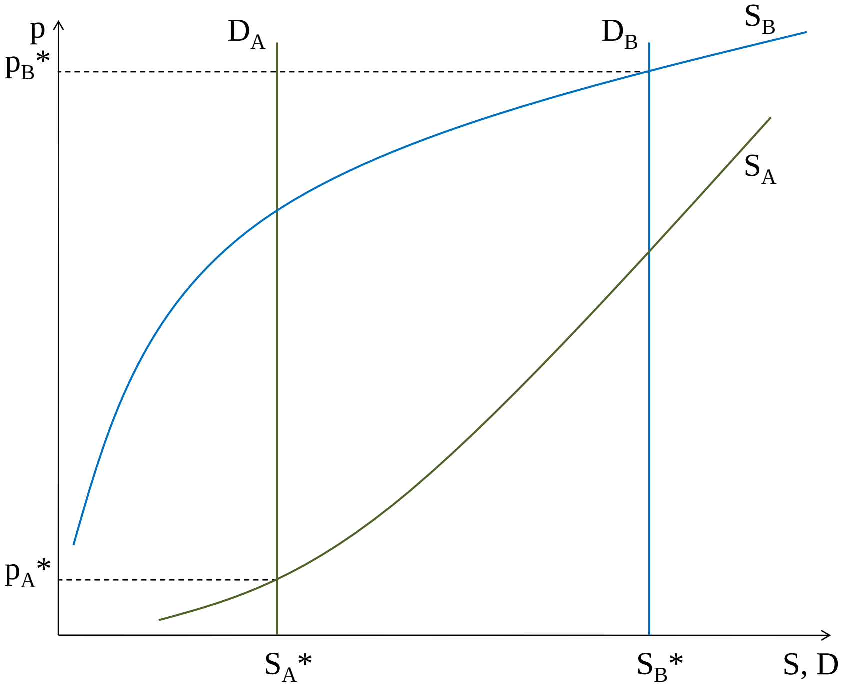

Figure 2: A stronger, but less elastic supply may lead to lower trading volumes.

In the example above, supply in market A is stronger but less elastic than supply in market B. Through the informational feedback on beliefs about average quality, demand indeed becomes so weak in market A that the trading volume decreases despite the increase in supply. Thus, the comparative statics of volumes is ambiguous unless strength and elasticity of supply move in the same direction.

A second example will be used to show that the conditions of Theorem 3 are not strong enough to make predictions on the comparative statics of prices.

Example 2. Consider cumulative functions $F_A(q) = q$ and $F_B(q) = q - q^2/9$, on the common support interval $[0, 1]$. Supply is clearly stronger in market A than in market B (cf. Figure 3). Elasticities of supply are given by $\varepsilon_A(p) = 1$ and $\varepsilon_B(p) = (9 - 2p)/(9 - p)$, respectively. In particular, elasticities of supply are non-increasing in both markets, with $\varepsilon_A(p) \geq \varepsilon_B(p)$ for $p \in (0, 1)$. The distribution of buyers' types is assumed identical across markets and uniform on the interval $[t_0, t_1] = [1.8, 2.8]$. Then, in market A, demand is perfectly inelastic with $D_A(p) = 0.8$, so that $p_A^* = S_A^* = 0.8$. In market B, however, average quality is

$$\bar{q}_B(p) = \frac{p}{6} \frac{27 - 4p}{9 - p}, \tag{25}$$

so that demand in market B reads

$$D_B(p) = 2.8 - 6 \left(\frac{9 - p}{27 - 4p} \right). \tag{26}$$

Equating supply and demand yields the equilibrium (p_B^*, S_B^*) with $p_B^* \approx 0.804$ and $S_B^* \approx 0.732$. In particular, $p_B^* > p_A^*$.

Still another example, where a stronger and more elastic supply leads to a higher price, will be presented below.[10]

Next, we examine the impact of a change in quality distribution on equilibrium prices. The results for prices are structurally similar to those obtained for volumes. However, the assumptions that need to be made differ decisively. To decrease prices, a stronger supply needs to become less elastic, while the "willingness to buy" should fall. Thus, we will assume that $\varepsilon_A^S(p) \leq \varepsilon_B^S(p)$ in the relevant domain, and also that comparison (19) concerning the buyers' preference parameters is satisfied with reversed inequality sign. The complete formal statement is the following.

Theorem 4. In the model with homogeneous sellers, if

(i) Condition (NIES) holds in both markets,

[10] See Example 3.

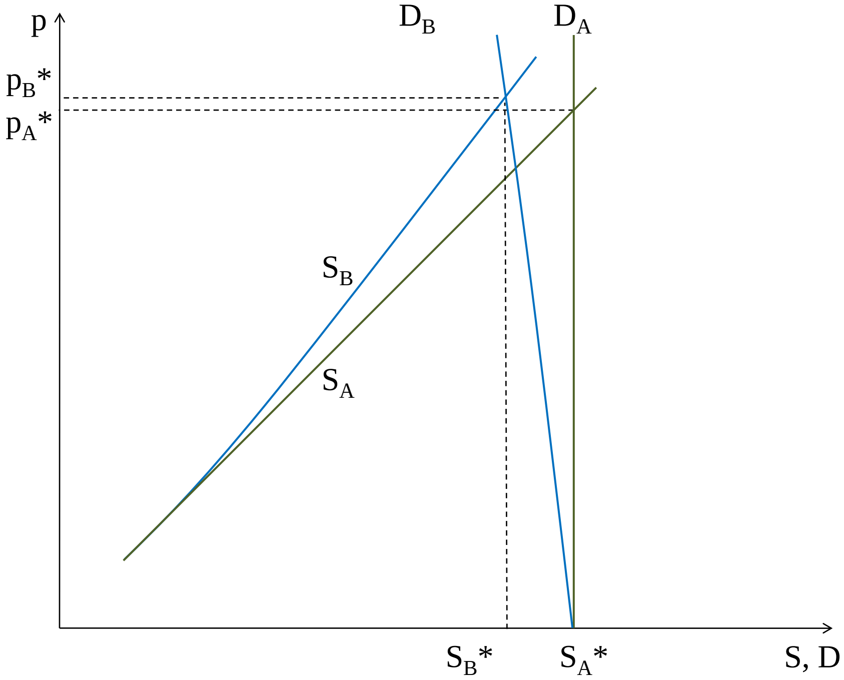

Figure 3: A stronger and more elastic supply may lead to lower prices.

(ii) $F_A(q) \geq F_B(q)$ for all q,

(iii) $\varepsilon_A^S(p) \leq \varepsilon_B^S(p)$ for all $p > q_0$, and

(iv) $\int_t^{t_1} h_A(\tau)d\tau \leq \int_t^{t_1} h_B(\tau)d\tau$ for all $t \leq t_1$,

then $p_A^* \leq p_B^*$, where p_i^* denotes the respective price in the unique equilibrium in market $i = A, B$.

Proof. To provoke a contradiction, we assume $p_B^* < p_A^*$. Using the fact that S_A is non-decreasing, and condition (ii) in the statement of the theorem, it follows that

$$S_A(p_A^*) \geq S_A(p_B^*) = F_A(p_B^*) \geq F_B(p_B^*) = S_B(p_B^*). \tag{27}$$

Hence, $S_A^* \geq S_B^*$. On the other hand, since D_A is non-increasing,

$$S_A^* = D_A(p_A^*) \leq D_A(p_B^*) \leq D_B(p_B^*) = S_B^*, \tag{28}$$

where the second inequality follows from conditions (i) and (iv) in the statement of the theorem. Indeed, because of condition (iv), it suffices to show that

$$\frac{p}{\bar{q}_A(p)} \geq \frac{p}{\bar{q}_B(p)} \tag{29}$$

for $p = p_B^*$. By Lemma A.1 in the Appendix, with the role of markets A and B exchanged, $\varepsilon_A^S(p) \leq \varepsilon_B^S(p)$ for all $p > q_0$ is a sufficient condition. Combining (27) and (28) yields $S_A^* = S_B^*$. But then, necessarily, all the weak inequalities in (27) and (28) must be equalities. In particular, $S_A(p_A^*) = S_A(p_B^*)$ and $D_A(p_A^*) = D_A(p_B^*)$. But excess demand is either strictly decreasing or negative. Hence, $p_A^* = p_B^*$, which contradicts the assumption made at the beginning of the proof. The assertion of the theorem follows. □

Intuitively, when supply is everywhere less elastic in market A than in market B, then the price-quality ratio $p/\bar{q}_A$ in market A is always larger or equal to the price-quality ratio $p/\bar{q}_B$ in market B. Therefore, demand in market A is everywhere at most as large as demand in market B. Thus, ceteris paribus, the equilibrium price is decreasing with a stronger and less elastic supply.[11]

The following example shows that condition (iii) in the statement of Theorem 4 is indeed required to obtain the result.

Example 3. Consider $F_A(q) = q$ and $F_B(q) = q - q^2/4$, defined on the common support interval $[0, 1]$. In particular, supply is stronger in market A than in market B. Elasticities of supply are non-increasing in both markets and defined as $\epsilon_A(p) = 1$ and $\epsilon_B(p) = 2(2 - p)/(4 - p)$ such that $\epsilon_A(p) \geq \epsilon_B(p)$ for $p \in (0, 1)$. In both markets, buyers' types are uniformly distributed on the interval $[t_0, t_1] = [1.8, 2.8]$.

[11] Since the equilibrium price corresponds to the highest quality offered in the market, a similar conclusion holds for the highest quality traded (yet not for the average quality).

Given these specifications, demand is constant in market A at $D_A(p) = 0.8$, hence $p_A^* = S_A^* = 0.8$ (cf. Figure 4). Demand in market B reads

$$D_B(p) = 2.8 - \frac{12 - 3p}{6 - 2p}. \tag{30}$$

Equating supply and demand yields the equilibrium (p_B^*, S_B^*) with $p_B^* \approx 0.776$ and $S_B^* \approx 0.625$. In particular, $p_B^* < p_A^*$.

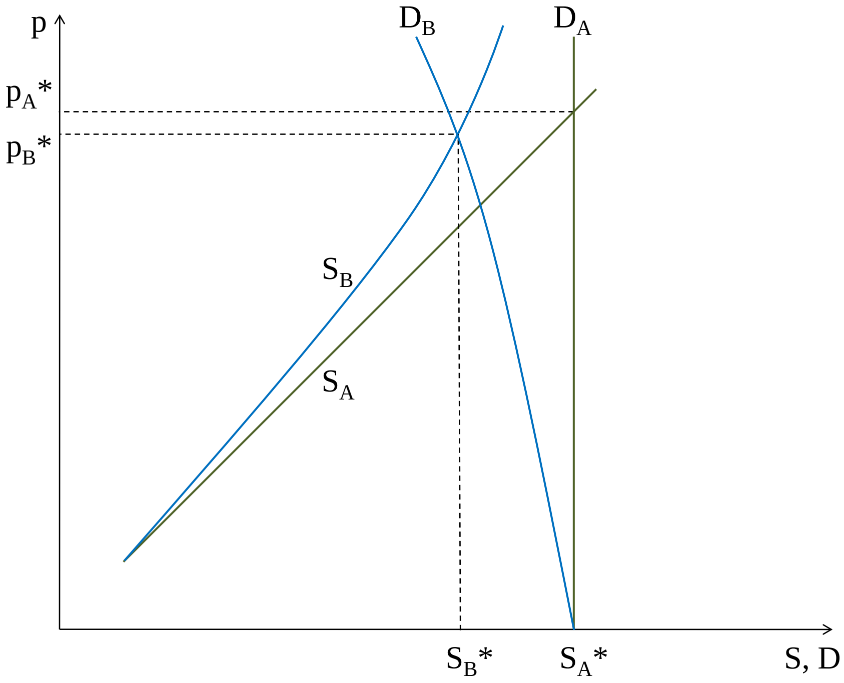

Figure 4: A stronger and more elastic supply may lead to higher prices.

In Example 3, supply is both stronger and more elastic in market A than in market B. There is an informational feedback on beliefs about average quality which increases demand in market A such that equilibrium price is higher in market A than in market B. We conclude from this example that comparative statics of prices is in general ambiguous unless the elasticity condition (iii) in Theorem 4 holds.

A final example will be used to illustrate that the conditions of Theorem 4 have no analogous implications for the comparative statics of volumes.

Example 4. Consider $F_A(q) = 2.5q - q^2$ and $F_B(q) = q - q^2/4$ defined on the common interval $[0, 1]$. Thus, $F_A(q) \geq F_B(q)$ for all $q > 0$ and $\varepsilon_A^S(p) \leq \varepsilon_B^S(p)$ where both elasticities of supply are non-increasing. Assuming a "willingness to buy" uniformly distributed on the interval $[t_0, t_1] = [1.8, 2.8]$ in both markets, condition (iv) in Theorem 4 is also satisfied. Then, Walrasian demand in market A and B are

$$D_A(p) = 2.8 - \frac{2.5 - p}{1.25 - 2/3p} \tag{31}$$

and

$$D_B(p) = 2.8 - \frac{12 - 3p}{6 - 2p}. \tag{32}$$

Equating supply and demand in both markets, one obtains the equilibria $(p_A^*, S_A^*) = (0.32, 0.697)$ and $(p_B^*, S_B^*) = (0.776, 0.625)$. The two outcomes are illustrated in Figure 5.

In Example 4, the equilibrium in market A with a stronger and less elastic supply features a higher trade volume. The case where trade volume declines in this situation is captured by Example 1.

4 Robustness: Heterogeneous sellers

So far, we have worked under the somewhat heroic assumption that sellers have identical preferences concerning quality. In a richer model, a seller is characterized by a pair $(q; \tilde{q})$, where q is the seller's reservation value, as before, and $\tilde{q}$ is the quality of the seller's endowment. In fact, this extension has been suggested by Wilson (1980, footnote 2). The present section will be used to show that the main results of Sections 2 and 3 carry over to this more general set-up without significant alterations of the assumptions.

To capture the extension formally, assume a two-dimensional continuous den-

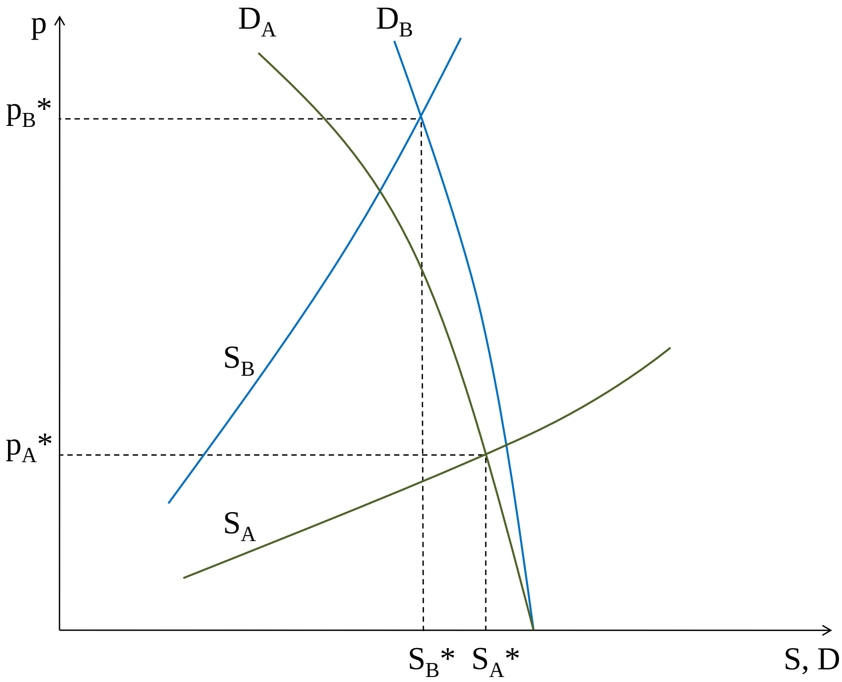

Figure 5: A stronger and less elastic supply may lead to higher trade volume

sity function ϕ on $[q_0, q_1] \times [\tilde{q}_0, \tilde{q}_1]$, where $\tilde{q}_0 < \tilde{q}_1$. The symbol f will be used to denote the density of the marginal distribution of reservation values on the interval $[q_0, q_1]$. This convention allows us to use the notation introduced in Section 2 also in the generalized set-up. In particular, $S(p)$ will be the market supply at price p, as before, etc.

The analysis of the outlined extension is simplified by the following consideration. Just as in the basic set-up, only sellers with a reservation price $q \leq p$ will rationally offer their respective endowment at a market price p. But given that buyers are risk-neutral, Walrasian demand depends only on the expected quality in the market, and not on the higher moments of the distribution. Therefore, to analyze the change in average quality implied by, say, a marginal increase in the market price p, it is sufficient to know only the *marginal quality* $\vartheta(p)$, where

$$\vartheta(p) = E[\tilde{q}|q = p] = \frac{\int_{\tilde{q}_0}^{\tilde{q}_1} \tilde{q}\phi(p, \tilde{q})d\tilde{q}}{\int_{\tilde{q}_0}^{\tilde{q}_1} \phi(p, \tilde{q})d\tilde{q}}. \tag{33}$$

Intuitively, the marginal quality is the average quality of the inflow of sellers caused by a marginal increase in the market price.

We are now ready to discuss more specifically the additional assumptions that are needed to generalize our results concerning uniqueness and comparative statics to the case of heterogeneous sellers.

In the case of uniqueness, it suffices to impose the following additional assumption.

(EMQ) $\partial \log \vartheta(p)/\partial \log q \leq 1$ for all $q \in (q_0, q_1)$.

This condition says that the price elasticity of marginal quality does not exceed one. Intuitively, the problem with an overly elastic marginal quality is that even if the elasticity of supply is non-increasing, the price-quality ratio might still fall with the market price, perverting the law of demand. The following generalization of Theorem 1 can be obtained.

Theorem 5. Assume that conditions (NIES) and (EMQ) hold in the model with heterogeneous sellers. Then $\partial D(p)/\partial p \leq 0$ for all $p \in (q_0, q_1)$, and the competitive equilibrium is unique.

Proof. It suffices to show (cf. the proof of Theorem 1) that the price-quality ratio is weakly increasing in the market price, i.e.

$$\frac{\partial}{\partial p}\left[\frac{pF(p)}{\int_{q_0}^{p} \vartheta(q)f(q)dq}\right] \geq 0 \tag{34}$$

for all $p \in (q_0, q_1)$. Rewriting inequality (34) yields

$$\frac{\partial}{\partial p}\left[\frac{\int_{q_0}^{p} qf(q) + F(q)dq}{\int_{q_0}^{p} \vartheta(q)f(q)dq}\right] \geq 0. \tag{35}$$

Write

$$\widetilde{g}(q,a) = (1-a)\vartheta(q)f(q) + a(qf(q) + F(q)) \tag{36}$$

for $a = 0, 1$. Then inequality (35) amounts to the log-supermodularity of

$$\widetilde{G}(p,a) = \int_{q_0}^{p} \widetilde{g}(q,a)dq \tag{37}$$

in (p, a). As log-supermodularity is stable under integration, a sufficient condition for inequality (34) is that

$$\frac{\partial}{\partial q}\left[\frac{qf(q) + F(q)}{\vartheta(q)f(q)}\right] \geq 0, \tag{38}$$

or equivalently, that

$$\frac{\partial}{\partial q}\left[\frac{q}{\vartheta(q)}\left(1 + \frac{1}{\varepsilon^s(q)}\right)\right] \geq 0. \tag{39}$$

Rewriting (39) delivers

$$\frac{\partial \varepsilon^s(q)/\partial q}{\varepsilon^s(q)(1+\varepsilon^s(q))} \leq \frac{1}{q}\left(1 - \frac{\partial \log \vartheta}{\partial \log q}\right). \tag{40}$$

With assumption (NIES) in place, the left-hand side of inequality (40) is non-positive. Therefore, to satisfy this inequality, it suffices to impose condition (EMQ). This proves the assertion, and therefore the theorem. □

The proof reveals that there is a trade-off between the elasticity of supply and the elasticity of marginal quality. Conditions (NIES) and (EMQ) separate this trade-off into two independent conditions.

Next, we look at comparative statics in the generalized model, assuming uniqueness. As before, the main issue is to ensure that the exogenous change on the supply side, here in the density ϕ, leads to a uniform response of Walrasian demand across all price levels. The additional assumption needed imposes restrictions on marginal quality functions ϑ_A and ϑ_B in markets A and B. The additional requirement is needed to make sure that an increase, say, of the elasticity of supply leads to a decline in the price-quality ratio. The following two theorems generalize Theorems 3 and 4.

Theorem 6. In the model with heterogeneous sellers, if

(i) the assumptions of Theorem 3 are satisfied,

(ii) condition (EMQ) holds in both markets,

(iii) $\vartheta_A(q) \geq \vartheta_B(q)$ for all $q \in (q_0, q_1)$, and

(iv) $\frac{\partial}{\partial q}\vartheta_i(q) \geq 0$ for all $q \in (q_0, q_1)$, for at least one market $i \in A, B$,

then $S_A^* \geq S_B^*$.

Proof. With view on the argument used in the proof of Theorem 3, it clearly suffices to show that average quality at any given price $p \in (q_0, q_1)$ is weakly better in market A than in market B, i.e., that

$$\frac{\int_{q_0}^{p} \vartheta_A(q) f_A(q) dq}{F_A(q)} \geq \frac{\int_{q_0}^{p} \vartheta_B(q) f_B(q) dq}{F_B(q)}. \tag{41}$$

To prove (41), assume first that $\partial\vartheta_B(q)/\partial q \geq 0$ for all $q \in (q_0, q_1)$. It follows then from inequality (50) in the proof of Lemma A.1 that

$$\frac{F_A(p')}{F_A(p)} \frac{\partial}{\partial q}\vartheta_B(q) \leq \frac{F_B(p')}{F_B(p)} \frac{\partial}{\partial q}\vartheta_B(q). \tag{42}$$

Hence, via integration over the interval $[q_0, p]$, we find

$$\frac{1}{F_A(p)} \int_{q_0}^{p} F_A(q) \frac{\partial}{\partial q}\vartheta_B(q) dq \leq \frac{1}{F_B(p)} \int_{q_0}^{p} F_B(q) \frac{\partial}{\partial q}\vartheta_B(q) dq. \tag{43}$$

Applying partial integration to (43) and subsequently exploiting condition (iii) in the statement of the theorem delivers indeed (41), as desired. Assume now that $\partial\vartheta_A(q)/\partial q \geq 0$ for all $q \in (q_0, q_1)$. The argument runs in this case precisely as before, so that we omit the details. This proves the assertion and thereby the theorem. □

Theorem 7. In the model with heterogeneous sellers, if

(i) the assumptions of Theorem 4 are satisfied,

(ii) condition (EMQ) holds in both markets,

(iii) $\vartheta_A(q) \leq \vartheta_B(q)$ for all $q \in (q_0, q_1)$, and

(iv) $\frac{\partial}{\partial q}\vartheta_i(q) \geq 0$ for all $q \in (q_0, q_1)$, for at least one market $i \in A, B$,

then $p_A^* \leq p_B^*$.

Proof. Similar to the proof of the previous theorem, it suffices to show that

$$\frac{\int_{q_0}^{p} \vartheta_A(q) f_A(q) dq}{F_A(q)} \leq \frac{\int_{q_0}^{p} \vartheta_B(q) f_B(q) dq}{F_B(q)} \tag{44}$$

for all $p \in (q_0, q_1)$. The only difference to (41) is the reversed inequality sign. The proof is analogous to that of Theorem 6. The details are omitted. □

5 Application: Interbank markets

In the aftermath of the global liquidity crisis 2007-2009, the analysis of interest rate levels and trade volumes in interbank money markets has received increasing attention both by academics and policy makers. At the center of the discussion stands the so-called *term money market* segment, where unsecured interbank loans are granted for a term of between one month and a year. On the empirical side, an interesting debate (see Taylor and Williams, 2009, McAndrews et al., 2008, and Wu, 2008) has developed in particular about the economic determinants of elevated interest rate spreads of term rates over the overnight benchmark, and relatedly, on the impact of innovative policy measures on market conditions. In this section, we will use our comparative statics results to elaborate on these issues from a theoretical perspective.

The changes that need to be made to the formal framework are straightforward and mostly in terms of interpretation.[12] On a more technical level, the route to translate the money market into the traditional Akerlof-Wilson perspective is to consider credit risk as an "economic bad" which necessitates a negative price, i.e., the interest payment. As a consequence of that change in perspective, the respective roles of demand and supply are exchanged. In particular, borrowers on the demand side are informed, while lenders on the supply side are uninformed. Apart from these superficial changes, however, the model remains the same. Therefore, to avoid a superfluous duplication of notation, we will describe merely the utility functions of borrowers and lenders, respectively, hoping the reader finds this information sufficiently comprehensive to follow through the details of the application.

The borrower can either seek credit in the interbank market or use an individual outside option. The interest rate of that outside option is private information to the borrower. If the interest rate of the outside option is higher than the interbank rate, the borrower seeks credit in the interbank market. Formally, each borrower maximizes the utility function

$$U_m = U_m(r, n \mid r^B) = (1-n)(-r^B) + n(-r), \tag{45}$$

where n is a dummy variable with $n = 1$ if the borrower finds credit in the interbank market and $n = 0$ if the borrower uses the outside option. The interest rate of the individual outside option is r^B, while r represents the competitive interbank

[12] Indeed, one of Akerlof's (1970) examples is the credit market in underdeveloped countries, which should share some characteristics with a distressed credit market in developed countries.

rate. To keep the changes to the model at a minimum, we impose the following simplifying assumption:[13]

Assumption 1. A lender's expected costs of default from a loan to a borrower are perfectly correlated with the borrower's opportunity funding cost r^B.

Each lender can either invest liquidity in the interbank market or use an outside option. The lenders know the distribution of r^B and correctly anticipate the average opportunity funding costs $\bar{r}^B$ of borrowers in the market. If the outside option's return is lower than the expected return from offering credit in the interbank market (interbank rate minus the expected default costs), lenders offer credit in the interbank market. Thus, a lender of type t maximizes expected utility

$$U_m^e = U_m^e(r, n \mid t, \bar{r}^B) = (1-n)r^L + n[r - t\bar{r}^B], \tag{46}$$

where r^L is the interest rate of the lender's outside option and t measures the relative valuation of default costs in terms of yields from investing liquidity in the interbank market.

Making straightforward replacements in the theoretical results, two main policy implications follow.

A first implication relates to the question of how a central bank can ensure a decline of elevated term spreads. In a normal market environment not affected by adverse selection, a simple liquidity injection is clearly sufficient. However, in a "lemons" market, any liquidity-providing monetary policy measure has also implications for the quality of borrowers remaining in the market. Specifically, Theorem 4 implies that a money market with lower demand for unsecured credit (and a stronger "willingness to lend") is more likely to feature lower interbank rates only if demand becomes more elastic. Therefore, to ease market conditions under adverse selection, the central bank should seek to inject liquidity so that residual

[13] The results obtained in Section 4 suggest, however, that the conclusions derived in the sequel are of a robust nature.

demand becomes more elastic than actual demand. The most natural way to ensure this is to direct funding liquidity in such a way that it reaches most likely those banks who need it most, e.g., through a competitive auction mechanism with wide access. This consideration actually supports the competitive design of the Federal Reserve's Term Auction Facility. The facility should have made borrowers less dependent on the money market and more responsive to market conditions, while lenders might have become more willing to lend in view of a future funding alternative.[14]

The second policy implication that can be drawn from the paper is somewhat more speculative by addressing the intricate issue of how to revive money markets. The Akerlof-Wilson model is consistent with trade volume either increasing or decreasing through a liquidity injection. Thus, according to our theory, a liquidity injection offers no guarantee whatsoever that credit markets "jumpstart" into working again. To revive interbank markets, our results rather suggest that demand should be made both stronger and more elastic at the same time. Also supply should be made to increase. Not all central bank instruments are equally suitable for this purpose, however. Regular open market operations and central bank facilities can only lower either demand or supply, and are therefore not directly useful. What our theory suggests instead is to lower reserve requirements, while providing the neutral amount of liquidity in a way that the most needy banks obtain it. This should induce cash-rich banks to deposit their funds in the market, i.e., supply should increase.[15] Also demand should increase since aggregate reserves in the banking systems are kept constant. Finally, demand should be more elastic as a consequence of the use of highest-price auctions, which provides an

[14]Empirical research supports this perspective. McAndrews et al. (2008), for instance, examine the impact of TAF on the Londoner Interbank Offered Rate during 2007 and 2008 and find that this facility, which amounted to a single-price auction with constraints on the maximum allotment per bidder, indeed helped to reduce the level of the market rates, consistent with our results.

[15]Incentives would be stronger with a lowered central bank deposit rate and/or a penalty rate applied to excess reserves.

outside option for the banks with the highest default risk.

6 Concluding remarks

This study has dealt with equilibrium uniqueness and monotone comparative statics in the Akerlof-Wilson model. A non-increasing elasticity of supply is sufficient to guarantee a unique equilibrium in a "lemons" market. The elasticity condition is tantamount to the requirement that the quality distribution is the multiplicative version of an arbitrary log-concave distribution. When supply becomes both stronger and more elastic, trade volumes increase unambiguously, while prices may rise or fall. On the other hand, when supply becomes both stronger and less elastic, then prices decrease, while volumes may either rise or fall. The elasticity conditions in these latter two results cannot be dropped without losing the monotone comparative statics property. All the results can be extended in a natural way to the case of a seller population with heterogeneous preferences regarding quality.

An application to the case of interbank money markets led to two surprisingly clear-cut policy implications. First, under adverse selection, pressure on market rates is more likely to be alleviated when competitive auction mechanisms with wide access are employed by the central bank. Second, to revive a money market, the provision of additional liquidity might actually be counterproductive. Instead, our theory hints towards a combination of measures, including a lowering of required reserves.

A Appendix

The following lemma is used in the proofs of Theorems 3 and 4.

Lemma A.1. Assume $\varepsilon_A(p) \geq \varepsilon_B(p)$ for all $p > q_0$. Then $\bar{q}_A(p) \geq \bar{q}_B(p)$ for all $p \geq q_0$.

Proof. Assume $\varepsilon_A(p) \geq \varepsilon_B(p)$ for all $p \in (q_0, q_1)$, i.e.

$$\frac{\partial \log F_A}{\partial \log p} \geq \frac{\partial \log F_B}{\partial \log p}. \tag{47}$$

Since $\partial \log p = (1/p)\partial p$, this implies

$$\frac{\partial \log F_A}{\partial p} \geq \frac{\partial \log F_B}{\partial p}. \tag{48}$$

Integrating (48) over the interval $[p', p]$, for $q_0 < p' < p$, yields

$$\log F_A(p) - \log F_A(p') \geq \log F_B(p) - \log F_B(p'). \tag{49}$$

Applying the negative exponential function to both sides of inequality (49), we obtain

$$\frac{F_A(p')}{F_A(p)} \leq \frac{F_B(p')}{F_B(p)}. \tag{50}$$

Hence, by another integration, one finds

$$\frac{1}{F_A(p)} \int_{q_0}^{p} F_A(q) dq \leq \frac{1}{F_B(p)} \int_{q_0}^{p} F_B(q) dq. \tag{51}$$

On the other hand, $\bar{q}_A(p) \geq \bar{q}_B(p)$ can be rewritten as

$$\frac{\int_{q_0}^{p} q f_A(q) dq}{F_A(p)} \geq \frac{\int_{q_0}^{p} q f_B(q) dq}{F_B(p)}. \tag{52}$$

Via partial integration, inequalities (51) and (52) can be seen to be equivalent. This proves the assertion. $\square$

References

Akerlof, G.A., 1970, The market for "lemons": quality uncertainty and the market mechanism, Quarterly Journal of Economics 84, 488-500.

Athey, S., 2002, Monotone comparative statics under uncertainty, Quarterly Journal of Economics 117, 187-223.

Bagnoli, M. and T. Bergström, 2006, Log-concave probability and its applications, in: C.D. Aliprantis et al., eds., Rationality and equilibrium: a symposium in honor of Marcel K. Richter, Studies in Economic Theory 26, 217-241.

Bigelow, J.P., 1990, Efficiency and adverse selection, Journal of Economic Theory 52, 380-405.

Genesove, D., 1993, Adverse selection in the wholesale used car market, Journal of Political Economy 101, 644-665.

Gibbons, R. and L.F. Katz, 1991, Layoffs and lemons, Journal of Labor Economics 9, 351-380.

Greenwald, B.C., 1986, Adverse selection in the labour market, Review of Economic Studies 53, 325-347.

Hendel, I. and A. Lizzeri, 1999, Adverse selection in durable goods markets, American Economic Review 89, 1097-1115.

Kessler, A.S., 2001, Revisiting the lemons market, International Economic Review 42, 25-41.

Kim, J.-C., 1985, The market for "lemons" reconsidered: a model of the used car market with asymmetric information, American Economic Review 75, 836-843.

Levin, J., 2001, Information and the market for lemons, RAND Journal of Economics 32, 657-666.

McAndrews, J., A. Sarkar, and Z. Wang, 2008, The effect of the term auction facility on the London inter-bank offered rate, Federal Reserve Bank of New York Staff Reports 335, 1-25.

Rose, C., 1993, Equilibrium and adverse selection, RAND Journal of Economics 24, 559-569.

Taylor, J.B. and J.C. Williams, 2009, A black swan in the money market, American Economic Journal: Macroeconomics 1, 58-83.

Wilson, C.A., 1979, Equilibrium and adverse selection, American Economic Review 69, 313-317.

Wilson, C.A., 1980, The nature of equilibrium in markets with adverse selection, Bell Journal of Economics 11, 108-130.

Wu, T., 2008, On the effectiveness of the Federal reserve's new liquidity facilities, Research Department, Federal Reserve Bank of Dallas.